UNCLASSIFIED

AD 201440

Armed Services Technical Information Agency

ARLINGTON HALL STATION
ARLINGTON 12 VIRGINIA

FOR
MICRO-CARD
CONTROL ONLY

1 OF 1

NOTICE: WHEN GOVERNMENT OR OTHER DRAWINGS, SPECIFICATIONS OR OTHER DATA ARE USED FOR ANY PURPOSE OTHER THAN IN CONNECTION WITH A DEFINITELY RELATED GOVERNMENT PROCUREMENT OPERATION, THE U. S. GOVERNMENT THEREBY INCURS NO RESPONSIBILITY, NOR ANY OBLIGATION WHATSOEVER; AND THE FACT THAT THE GOVERNMENT MAY HAVE FORMULATED, FURNISHED, OR IN ANY WAY SUPPLIED THE SAID DRAWINGS, SPECIFICATIONS, OR OTHER DATA IS NOT TO BE REGARDED BY IMPLICATION OR OTHERWISE AS IN ANY MANNER LICENSING THE HOLDER OR ANY OTHER PERSON OR CORPORATION, OR CONVEYING ANY RIGHTS OR PERMISSION TO MANUFACTURE, USE OR SELL ANY PATENTED INVENTION THAT MAY IN ANY WAY BE RELATED THERETO.

UNCLASSIFIED

16 May 1958

Report No. DA-TR-9-58
Copy No. A [illegible]

BALLISTIC MISSILE AGENCY

HUNTSVILLE, ALABAMA

DAMPED FLUID OSCILLATIONS IN A CIRCULAR CYLINDRICAL TANK DUE TO BENDING OF TANK WALL (U)

FILE COPY

Return to

ASTIA

ARLINGTON HALL STATION

ARLINGTON 12, VIRGINIA

Attn: TISSS

AD No. 201440

ASTIA FILE COPY

BEST

AVAILABLE

COPY

REPORT NO. DA-TR-9-58

ARMY BALLISTIC MISSILE AGENCY
REDSTONE ARSENAL, ALABAMA
DEVELOPMENT OPERATIONS DIVISION
AEROBALLISTICS LABORATORY
DYNAMICS ANALYSIS BRANCH

DA TECHNICAL REPORT NO. 9-58

16 MAY 1958

DAMPED FLUID OSCILLATIONS IN A CIRCULAR CYLINDRICAL TANK DUE TO BENDING OF TANK WALL (U)

[illegible]

[illegible]:

Helmut F. Bauer
[illegible]
Research Engineer

APPROVED BY:

F. W. Zetti ...
F. W. [illegible]
Chief, Vibration and Flutter Section

AHLuskin
for [illegible] J. [illegible]
Chief, Dynamics Analysis Branch

Geissler
E. D. [illegible]
Director, Aeroballistics Laboratory

LIST OF FIGURES

CONTENTS

I. SUMMARY

The effect of bending tank walls on the fluid of a partially filled cylindrical tank is considered with damping. The following investigation applies the theory of Reference 3 with incorporated damping factors g. Physical reasons lead to the transformation of the theoretical formulae. The pressure distribution, fluid force, and fluid moment due to bending tank walls were determined and graphed. For low and very high frequencies, the damping is of minor importance. Near the resonances, the damping is of strong influence.

The obtained results will be incorporated into the bending flutter analysis.

So far, in the bending flutter analysis, the mass of the propellants has been assumed as mass-points distributed along the missile axis. However, the energy dissipated in the oscillating liquid may cause a considerable increase in the effective structural damping of the missile, thus permitting greater freedom in the lay-out of the electrical control system. At low frequencies ("first sloshing mode"), the bending of the missile may change the character of the oscillation sufficiently to make the incorporation of the missile bending in the investigation of these frequencies very desirable.

II. LIST OF SYMBOLS

r, φ, z	Cylindrical Coordinates
t [sec]	Time
p $[kg\ m^{-2}]$	Pressure
ρ $[kg\ sec^2 m^{-4}]$	Fluid density
a [m]	Tank radius
h [m]	Fluid height
$w_B(z)$ [m]	Bending Function (small displacement in x-direction)
ω_n $[sec^{-1}]$	Natural circular frequency of rigid tank
ω $[sec^{-1}]$	Circular forced frequency
$\ddot{b}$ $\left[\frac{m}{sec^2}\right]$	Acceleration in x-direction
F [kg]	Fluid force perpendicular to tank wall
M [kg m]	Fluid moment around center of gravity of undisturbed fluid.
J_1	Bessel function of first order and first kind
ε_n	Zeros of $J_1'(\varepsilon_n) = 0$ $(n = 1, 2, 3......)$
m $\left[\frac{kg\ sec^2}{m}\right]$	Fluid mass
C_n	$= -\dfrac{2ia\omega}{(\varepsilon_n^2-1)\, J_1(\varepsilon_n)}$
λ_n	$= \dfrac{\varepsilon_n}{a}$ $\left[\frac{1}{m}\right]$
a_n, b_n	Constants, which values are determined by bottom and fluid surface boundary conditions.
ξ	damping factor

III. INTRODUCTION:

The exact solution of fluid motions in a cylindrical tank due to bending walls was solved in Reference 3 for ideal fluid (incompressible and non-viscous). The solution was found by solving the Poisson-Differential equation for special boundary conditions. The result, however, has singularities at the natural frequencies which are for the frequency of interest very close to each other. Therefore, the solution cannot be applied around these singularities. An approximation is obtained by introducing damping terms in the resonance form of the theoretical solution of the ideal fluid case as done in Reference 2. Physical considerations for infinite damping lead to transformation of the original formulae of Reference 3.

IV. PRESSURE DISTRIBUTION, FLUID FORCE AND FLUID MOMENT:

To introduce the damping factor in the different values of Reference 1, we have to consider some physical reasons which will lead us to the transformation of the formulae. For no damping, the transformed formulae has to be the formulae from Reference 3, while for infinite damping, the formulae has to tend to certain values given below.

Since in the flutter analysis we are only interested in the pressure distribution, fluid force and fluid moment, the values for these expressions will only be derived from Reference 3(41a). It is seen that the wall pressure distribution is:

$$P_{wall} = \rho e^{i\omega t} \cos\phi \left\{ a\omega^2 x_o(z) - i\omega \sum_{n=1}^{\infty} \left[a_n e^{\lambda_n z} + b_n e^{-\lambda_n z} + \frac{c_n}{2\lambda_n} \left(e^{\lambda_n z} I_- - e^{-\lambda_n z} I_+ \right) \right] I_1(\varepsilon_n) \right\} - b\rho z \qquad (1)$$

the fluid force (4)

$$F = m e^{i\omega t} \omega^2 \left\{ \frac{1}{h} \int_{-h}^{0} x_0(z)\,dz - \frac{i}{\omega a h} \sum_{n=1}^{\infty} \left[\frac{a_n}{\lambda_n}\left(1 - e^{-\lambda_n h}\right) - \frac{b_n}{\lambda_n}\left(1 - e^{\lambda_n h}\right) + \right.\right.$$

$$\left.\left. + \frac{C_n}{2\lambda_n}\left([I_1]_{-h}^{0} - [I_2]_{-h}^{0} \right) \right] I_1(\varepsilon_n) \right\} \qquad (2)$$

and the moment referred to the undisturbed position of the center of gravity

is: (50)

$$\vec{M} = m\omega^2 c^{i\omega t} \left\{ \frac{1}{2} \int_{-h}^{0} x_0(z)\,dz + \frac{1}{h} \int_{-h}^{0} z\, x_0(z)\,dz + \frac{a^2}{4h} x_0(-h) - \right.$$

$$- \frac{i}{2\omega a} \sum_{n=1}^{\infty} \left[\frac{a_n}{\lambda_n}\left(1 - \frac{2}{\lambda_n h} + \frac{2e^{-\lambda_n h}}{\lambda_n h} + e^{-\lambda_n h}\right) + \frac{b_n}{\lambda_n}\left(\frac{2e^{\lambda_n h}}{\lambda_n h} - \frac{2}{\lambda_n h} - e^{\lambda_n h} - 1\right) \right.$$

$$\left. + \frac{C_n}{2\lambda_n}\left([I_1]_{-h}^{0} - [I_2]_{-h}^{0} + \frac{2}{h}[I_3]_{-h}^{0} - \frac{2}{h}[I_4]_{-h}^{0} \right) \right] I_1(\varepsilon_n) -$$

$$\left. - \frac{i}{\omega} \sum_{n=1}^{\infty} \left[a_n e^{-\lambda_n h} + b_n e^{\lambda_n h} + \frac{C_n}{2\lambda_n}\left(e^{-\lambda_n h} I_-^{(-h)} - e^{\lambda_n h} I_+^{(-h)} \right) \right] \frac{I_1(\varepsilon_n)}{\varepsilon_n^2 \frac{h}{a}} \right\} \qquad (3)$$

where

$$I_- = \int x_0''(z)\, e^{-\lambda_n z}\, dz \qquad (4a)$$

$$I_+ = \int x_0''(z)\, e^{\lambda_n z}\, dz \qquad (4b)$$

$$[I_1]_{-h}^{0} = \int_{-h}^{0} \left[e^{\lambda_n z} \int x_0''(z)\, e^{-\lambda_n z}\, dz \right] dz \qquad (5a)$$

$$[I_2]_{-h}^{0} = \int_{-h}^{0} \left[e^{-\lambda_n z} \int x_0''(z)\, e^{\lambda_n z}\, dz \right] dz \qquad (5b)$$

$$[I_3]_{-h}^{0} = \int_{-h}^{0} \left[z e^{\lambda_n z} \int x_0''(z)\, e^{-\lambda_n z}\, dz \right] dz \qquad (5c)$$

$$[I_4]_{-h}^{0} = \int_{-h}^{0} \left[z e^{-\lambda_n z} \int x_0''(z)\, e^{\lambda_n z}\, dz \right] dz \qquad (5d)$$

Considering now an infinite damping of the fluid, the force on the tank

wall length dz is (inertia force)

$$q_n(z)\,dz = - dm\,\ddot{x} = \rho\pi a^2 \omega^2 x_0(z) e^{i\omega t} dz \qquad (6a)$$

the total force is then

$$F_{g=\infty} = m\omega^2 e^{i\omega t} \cdot \frac{1}{h}\int_{-h}^{0} x_0(z)\,dz \tag{6}$$

This is the first term in formula (2). The force finally can be written as:

$$F = m\omega^2 e^{i\omega t}\left\{\frac{1}{h}\int_{-h}^{0} x_0(z)\,dz - \frac{i}{\omega a h}\sum_{n=1}^{\infty}\left[\frac{a_n}{\lambda_n}\left(1-e^{-\lambda_n h}\right) - \right.\right. \tag{7}$$

$$\left.\left. - \frac{b_n}{\lambda_n}\left(1-e^{\lambda_n h}\right) + \frac{C_n\left(\frac{\omega_n^2}{\omega^2}-1\right)}{2\lambda_n\left(\frac{\omega_n^2}{\omega^2}-1+ig\frac{\omega_n}{\omega}\right)}\left([I_1]_{-h}^{0} - [I_2]_{-h}^{0}\right)\right] I_1(\varepsilon_n)\right\}$$

The pressure distribution is:

$$p_{wall} = \rho e^{i\omega t}\cos\phi\left\{a\omega^2 x_0(z) - i\omega\sum_{n=1}^{\infty}\left[a_n e^{\lambda_n z} + b_n e^{-\lambda_n z} + \right.\right.$$

$$\left.\left. + \frac{C_n\left(\frac{\omega_n^2}{\omega^2}-1\right)}{2\lambda_n\left(\frac{\omega_n^2}{\omega^2}-1+ig\frac{\omega_n}{\omega}\right)}\left(e^{\lambda_n z} I_- - e^{-\lambda_n z} I_+\right)\right] I_1(\varepsilon_n)\right\} - b\rho z \tag{8}$$

The moment of the fluid with infinite damping $g = \infty$ composes of two parts:

(a) moment due to inertia

(b) moment due to rotation.

The moment around the undisturbed center of gravity of a disc (See Fig. 1), due to inertia is:

$$q_n(z)\,dz \cdot \left(\frac{h}{2}+z\right)$$

which is integrated

$$\omega^2 m e^{i\omega t}\,\frac{1}{h}\int_{-h}^{0} x_0(z)\left(\frac{h}{2}+z\right)dz$$

The moment due to the rotation of the disc is:

$$-I\ddot{\theta} \qquad \text{(I is moment of inertia)}$$

The angle of rotation of the disc

$$\theta = \theta_0 e^{i\omega t} = x_0'(z) e^{i\omega t}$$

Since the slice is chosen very thin, the moment of inertia of it is:

$$I = \frac{dm\, a^2}{4}$$

with dm as the mass of the slice:

$$dm = \pi a^2 \rho\, dz$$

and integrated, the total moment due to rotation of the discs is:

$$-m\omega^2 e^{i\omega t} \frac{a^2}{4}\frac{1}{h}\int_{-h}^{0} x_0'(z)\,dz = m\omega^2 e^{i\omega t} \frac{a^2}{4h}\left[x_0(-h) - x_0(0)\right]$$

The moment around the undisturbed center of gravity for infinite damping is then:

$$\widetilde{M}_{g,\infty} = \omega^2 m e^{i\omega t}\Big\{\underbrace{\frac{1}{h}\int_{-h}^{0}\left(\frac{h}{2}+z\right)x_0(z)\,dz}_{\text{moment due to inertia}} + \underbrace{\frac{a^2}{4h}\left[x_0(-h) - x_0(0)\right]}_{\text{moment due to rotation}}\Big\} \tag{9}$$

The first two terms appear in formula (3). The moment, with damping, around the undisturbed position is then:

$$\begin{aligned}
\widetilde{M} = m\omega^2 e^{i\omega t}\Big\{ & \frac{1}{2}\int_{-h}^{0} x_0(z)\,dz + \frac{1}{h}\int_{-h}^{0} z\, x_0(z)\,dz + \frac{a^2}{4h}\left[x_0(-h) - x_0(0)\right] - \\
& - \frac{i}{2\omega a}\sum_{n=1}^{\infty}\Big[\frac{a_n}{\lambda_n}\left(1 - \frac{2}{\lambda_n h} + \frac{2e^{-\lambda_n h}}{\lambda_n h} + e^{-\lambda_n h}\right) + \\
& + \frac{b_n}{\lambda_n}\left(\frac{2e^{\lambda_n h}}{\lambda_n h} - \frac{2}{\lambda_n h} - e^{\lambda_n h} - 1\right) + \frac{C_n\left(\frac{\omega_n^2}{\omega^2} - 1\right)}{2\lambda_n\left(\frac{\omega_n^2}{\omega^2} - 1 + ig\frac{\omega_n}{\omega}\right)}\cdot\Big([I_1]_{-h}^{0} - [I_2]_{-h}^{0} + \\
& + \frac{2}{h}[I_3]_{-h}^{0} - \frac{2}{h}[I_4]_{-h}^{0}\Big)\Big] I_1(\varepsilon_n) - \frac{i}{\omega}\sum_{n=1}^{\infty}\Big[a_n e^{-\lambda_n h} + b_n e^{\lambda_n h} + \\
& + \frac{C_n\left(\frac{\omega_n^2}{\omega^2} - 1\right)}{2\lambda_n\left(\frac{\omega_n^2}{\omega^2} - 1 + ig\frac{\omega_n}{\omega}\right)}\left(e^{-\lambda_n h} I_1^{(-h)} - e^{\lambda_n h} I_2^{(-h)}\right)\Big]\frac{I_1(\varepsilon_n)}{\varepsilon_n^2 \frac{h}{a}} + \frac{2x_0(0)a^2}{h}\sum_{n=1}^{\infty}\frac{\left(\frac{\omega_n^2}{\omega^2} - 1\right)}{\varepsilon_n^2(\varepsilon_n^2 - 1)\left(\frac{\omega_n^2}{\omega^2} - 1 + ig\frac{\omega_n}{\omega}\right)}\Big\}
\end{aligned} \tag{10}$$

The a_n and b_n - values are taken for a motion of the tank bottom parallel to the x-direction and are (Compare 38a, b) in Reference 3):

$$a_n = \frac{C_n}{2\omega^2\cosh(\lambda_n h)\left(\frac{\omega_n^2}{\omega^2}-1+ig\frac{\omega_n}{\omega}\right)}\Big\{ b x_0'(0)e^{\lambda_n h} - x_0'(-h)\left[b+\frac{\omega^2}{\lambda_n}\right] - \tag{11a}$$

$$- \omega^2 x_0(0) e^{\lambda_n h} + I_-^{(0)}\frac{e^{\lambda_n h}}{2}\left[\frac{\omega^2}{\lambda_n}-b\right] - I_+^{(0)}\frac{e^{\lambda_n h}}{2}\left[\frac{\omega^2}{\lambda_n}+b\right] +$$

$$+ I_-^{(-h)}\frac{e^{-\lambda_n h}}{2}\left[b+\frac{\omega^2}{\lambda_n}\right] + I_+^{(-h)}\frac{e^{\lambda_n h}}{2}\left[b+\frac{\omega^2}{\lambda_n}\right]\Big\}$$

$$b_n = \frac{C_n}{2\omega^2\cosh(\lambda_n h)\left(\frac{\omega_n^2}{\omega^2}-1+ig\frac{\omega_n}{\omega}\right)}\Big\{ x_0'(0)\, b\, e^{-\lambda_n h} - \omega^2 x_0(0) e^{-\lambda_n h} -$$

$$- b x_0'(-h) + \frac{\omega^2}{\lambda_n}x_0'(-h) + I_-^{(0)}\frac{e^{-\lambda_n h}}{2}\left[\frac{\omega^2}{\lambda_n}-b\right] - \tag{11b}$$

$$- I_+^{(0)}\frac{e^{-\lambda_n h}}{2}\left[\frac{\omega^2}{\lambda_n}+b\right] + I_-^{(-h)}\frac{e^{-\lambda_n h}}{2}\left[b-\frac{\omega^2}{\lambda_n}\right] + I_+^{(-h)}\frac{e^{\lambda_n h}}{2}\left[b-\frac{\omega^2}{\lambda_n}\right]\Big\}$$

V. APPLICATION:

We choose now a tank with the length H and a radius a filled to the height h with fluid (as in Reference 3). The tank bottom and tank top are fixed and are not moving, while the tank wall oscillates in a parabolic shape.

The equation $x_0(z)$ of the bending tank wall is:

$$x_0(z) = -\frac{4\bar{x}_0 a}{\left(\frac{H}{a}\right)^2}\left\{\left(\frac{z}{a}\right)^2 + \left(\frac{z}{a}\right)\left(\frac{h}{a}-\frac{H}{a}\right) - \frac{h}{a}\left(\frac{H}{a}-\frac{h}{a}\right)\right\}$$

$$x_0(-h) = 0 \qquad , \qquad x_0(0) = \frac{4\bar{x}_0 a}{\left(\frac{H}{a}\right)^2}\left(\frac{H}{a}-\frac{h}{a}\right)\frac{h}{a}$$

Furthermore

$$X_0'(z) = -\frac{4\bar{x}_0}{\left(\frac{H}{a}\right)^2}\left[2\frac{z}{a} + 2\frac{h}{a} - \frac{H}{a}\right],$$

$$X_0'(0) = -\frac{4\bar{x}_0}{\left(\frac{H}{a}\right)^2}\left[2\frac{h}{a} - \frac{H}{a}\right],$$

$$X_0'(-h) = \frac{4\bar{x}_0}{\left(\frac{H}{a}\right)};$$

$$X_0''(z) = X_0''(0) = X_0''(-h) = -\frac{8\bar{x}_0}{a\left(\frac{H}{a}\right)^2};$$

$\bar{x}_0 a$ is the maximum displacement at the tank wall.

$\bar{x}_0$ is a pure number ($\bar{x}_0 \ll 1$)

Determination of the integrals

$$I_- = -\frac{8\bar{x}_0}{a\left(\frac{H}{a}\right)^2}\int e^{-\lambda_n z}dz = \frac{8\bar{x}_0}{\varepsilon_n\left(\frac{H}{a}\right)^2}e^{-\lambda_n z}$$

$$I_+ = -\frac{8\bar{x}_0}{a\left(\frac{H}{a}\right)^2}\int e^{\lambda_n z}dz = -\frac{8\bar{x}_0}{\varepsilon_n\left(\frac{H}{a}\right)^2}e^{\lambda_n z}$$

$$I_-^{(0)} = -I_+^{(0)} = \frac{8\bar{x}_0}{\varepsilon_n\left(\frac{H}{a}\right)^2}$$

$$I_-^{(-h)} = \frac{8\bar{x}_0}{\varepsilon_n\left(\frac{H}{a}\right)^2}e^{\lambda_n h}, \qquad I_+^{(-h)} = -\frac{8\bar{x}_0}{\varepsilon_n\left(\frac{H}{a}\right)^2}e^{-\lambda_n h};$$

$$I_1 = \frac{8\bar{x}_0}{\varepsilon_n\left(\frac{H}{a}\right)^2}z, \qquad I_2 = -\frac{8\bar{x}_0}{\varepsilon_n\left(\frac{H}{a}\right)^2}z,$$

$$\left[I_1\right]_{-h}^{0} = \frac{8\bar{x}_0 h}{\varepsilon_n\left(\frac{H}{a}\right)^2}, \qquad \left[I_2\right]_{-h}^{0} = -\frac{8\bar{x}_0 h}{\varepsilon_n\left(\frac{H}{a}\right)^2}$$

$$\left[I_3\right]_{-h}^{0} = -\frac{4\bar{x}_0 h^2}{\varepsilon_n\left(\frac{H}{a}\right)^2}, \qquad \left[I_4\right]_{-h}^{0} = \frac{4\bar{x}_0 h^2}{\varepsilon_n\left(\frac{H}{a}\right)^2}$$

The constants a_n and b_n (11a,b) are:

$$a_n = \frac{4\bar{x}_0 i\omega a^2}{\left(\frac{H}{a}\right)^2(\varepsilon_n^2-1)\,I_1(\varepsilon_n)\cosh\left(\varepsilon_n\frac{h}{a}\right)\left(\frac{\omega_n^2}{\omega^2}-1+ig\frac{\omega_n}{\omega}\right)}\cdot\left\{\frac{H}{a}\left[\frac{b}{a\omega^2}+\frac{1}{\varepsilon_n}\right]-\frac{2}{\varepsilon_n^2}e^{\lambda_n h}+\right.$$

$$\left.+e^{\lambda_n h}\left[\frac{b}{a\omega^2}\left(2\frac{h}{a}-\frac{H}{a}\right)+\frac{h}{a}\left(\frac{H}{a}-\frac{h}{a}\right)\right]\right\}$$

$$b_n = \frac{4\bar{x}_o i\omega a^2}{\left(\frac{H}{a}\right)^2(\varepsilon_n^2-1)I_1(\varepsilon_n)\cosh(\varepsilon_n\frac{h}{a})\left(\frac{\omega_n^2}{\omega^2}-1+ig\frac{\omega_n}{\omega}\right)}\left\{\frac{H}{a}\left[\frac{b}{a\omega^2}-\frac{1}{\varepsilon_n}\right]-\frac{2}{\varepsilon_n^2}e^{-\lambda_n h}+\right.$$

$$\left.+e^{-\lambda_n h}\left[\frac{b}{a\omega^2}\left(2\frac{h}{a}-\frac{H}{a}\right)+\frac{h}{a}\left(\frac{H}{a}-\frac{h}{a}\right)\right]\right\}$$

The wall pressure finally is:

$$p_{wall} = \frac{4\bar{x}_o a^2\omega^2\rho e^{i\omega t}\cos\phi}{\left(\frac{H}{a}\right)^2}\cdot\left\{\frac{h}{a}\left(\frac{H}{a}-\frac{h}{a}\right)+\frac{z}{a}\left(\frac{H}{a}-2\frac{h}{a}\right)-\left(\frac{z}{a}\right)^2+\right.$$

$$+2\sum_{n=1}^{\infty}\frac{\left[\frac{b}{a\omega^2}\left(2\frac{h}{a}-\frac{H}{a}\right)+\frac{h}{a}\left(\frac{H}{a}-\frac{h}{a}\right)-\frac{2}{\varepsilon_n^2}\right]\cosh\left[\frac{\varepsilon_n}{a}(z+h)\right]+\frac{H}{a}\frac{b}{a\omega^2}\cosh(\varepsilon_n\frac{z}{a})+\frac{H}{a\varepsilon_n}\sinh(\varepsilon_n\frac{z}{a})}{(\varepsilon_n^2-1)\cosh(\varepsilon_n\frac{h}{a})\left[\frac{\omega_n^2}{\omega^2}-1+ig\frac{\omega_n}{\omega}\right]}$$

$$\left.-4\sum_{n=1}^{\infty}\frac{\left(\frac{\omega_n^2}{\omega^2}-1\right)}{\varepsilon_n^2(\varepsilon_n^2-1)\left(\frac{\omega_n^2}{\omega^2}-1+ig\frac{\omega_n}{\omega}\right)}\right\} - b\rho z\,.$$

The Fluid Force is:

$$F = \frac{m\omega^2\bar{x}_o a e^{i\omega t}}{\left(\frac{H}{a}\right)^2}\left[\frac{2}{3}\frac{h}{a}\left\{3\frac{H}{a}-2\frac{h}{a}\right\}+\right.$$

$$+8\sum_{n=1}^{\infty}\frac{\left[\frac{2b}{a\omega^2}\frac{h}{a}+\frac{h}{a}\left(\frac{H}{a}-\frac{h}{a}\right)-\frac{2}{\varepsilon_n^2}\right]\tanh(\varepsilon_n\frac{h}{a})+\frac{\left(\frac{H}{a}\right)}{\varepsilon_n\cosh(\varepsilon_n h/a)}-\frac{H}{a\varepsilon_n}}{\left(\varepsilon_n\frac{h}{a}\right)(\varepsilon_n^2-1)\left(\frac{\omega_n^2}{\omega^2}-1+ig\frac{\omega_n}{\omega}\right)} -$$

$$\left.-16\sum_{n=1}^{\infty}\frac{\left(\frac{\omega_n^2}{\omega^2}-1\right)}{\varepsilon_n^2(\varepsilon_n^2-1)\left(\frac{\omega_n^2}{\omega^2}-1+ig\frac{\omega_n}{\omega}\right)}\right]$$

The moment referred to the undisturbed position of the center of gravity of the undisturbed fluid is:

$$\widetilde{M} = \frac{m\omega^2\bar{x}_o a^2 e^{i\omega t}}{\left(\frac{H}{a}\right)^2}\cdot\left[\left(\left(\frac{1}{3}\frac{h}{a}\right)^2-1\right)\left(\frac{H}{a}-\frac{h}{a}\right)+4\sum_{n=1}^{\infty}\frac{1}{\varepsilon_n(\varepsilon_n^2-1)\left(\frac{\omega_n^2}{\omega^2}-1+ig\frac{\omega_n}{\omega}\right)}\right.$$

$$\left\{\frac{\left[\frac{H}{a}\frac{1}{\varepsilon_n}-\frac{8}{\varepsilon_n^2}\frac{1}{\varepsilon_n\frac{h}{a}}+\frac{8}{\varepsilon_n}\frac{b}{a\omega^2}-\frac{6b}{a\omega^2}\frac{H}{a}\frac{1}{\varepsilon_n\frac{h}{a}}+\frac{4}{\varepsilon_n}\left(\frac{H}{a}-\frac{h}{a}\right)\right]}{\cosh\left(\varepsilon_n\frac{h}{a}\right)}\right. +$$

$$\tanh(\varepsilon_n\frac{h}{a})\left[\frac{2b}{a\omega^2}\frac{h}{a}-\frac{2b}{a\omega^2}\frac{H}{a}+\frac{h}{a}\left(\frac{H}{a}-\frac{h}{a}\right)-\frac{2}{\varepsilon_n^2}-\frac{4}{\varepsilon_n\frac{h}{a}}\frac{H}{a\varepsilon_n}\right]+$$

$$
+\left[\frac{4}{\varepsilon_n^4}\frac{1}{\varepsilon_n\frac{h}{a}}+\frac{6}{\varepsilon_n\frac{h}{a}}\frac{H}{a}\frac{b}{a\omega^2}+\frac{H}{a\varepsilon_n}-\frac{2}{\varepsilon_n}\left(\frac{H}{a}-\frac{h}{a}\right)-\frac{4}{\varepsilon_n}\frac{b}{a\omega^2}\right]\Bigg\}
$$

$$
-16\sum_{n=1}^{\infty}\frac{\left(\frac{\omega_n^2}{\omega^2}-1\right)}{\varepsilon_n^4\left(\varepsilon_n^2-1\right)\left(\varepsilon_n\frac{h}{a}\right)\left(\frac{\omega_n^2}{\omega^2}-1+ig\frac{\omega_n}{\omega}\right)}
$$

$$
+8\sum_{n=1}^{\infty}\frac{\left(\frac{H}{a}-\frac{h}{a}\right)\left(\frac{\omega_n^2}{\omega^2}-1\right)}{\varepsilon_n^4\left(\varepsilon_n^2-1\right)\left(\frac{\omega_n^2}{\omega^2}-1+ig\frac{\omega_n}{\omega}\right)}\Bigg]
$$

VI. CONCLUSION

The exact solution for fluid motion in a cylindrical tank due to bending tank walls is derived in Reference 3. for an ideal fluid (incompressible and nonviscous). The results at the natural frequencies of the fluid in the tank are irregular and cannot be used. Therefore, an approximation was obtained by introducing an imaginary damping term in the resonance terms as performed in Reference 2. Since the bending of a missile occurs at higher frequencies than gust and control disturbances, and since the maximum bending amplitude $\bar{X}_o a$ of the first mode may be of the order of a small translational amplitude, the values obtained for fluid oscillations due to bending tank walls will be of the magnitude of those for rigid tank walls. It is therefore impossible to neglect them in all cases.

It can be seen from the previous formulae that the influence of the maximum bending displacement $\bar{X}_o a$ of the tank wall is linear.

The absolute pressure

$$\left| \frac{p_{max} + bgz}{\bar{X}_o a^2 \rho e^{i\omega t} \cos\varphi} \right|$$

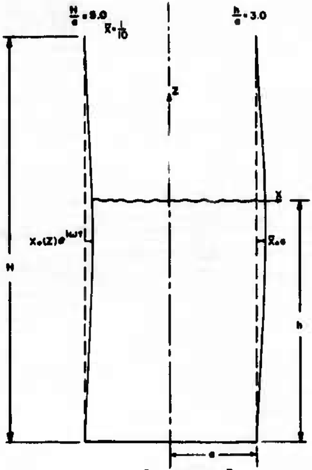
is graphed in Fig. 2, 3, and 4 for a fluid height h=2 and h=4 for different forced frequencies ω and damping factors g. For very small frequencies, the damping is only of very small influence. Near the lower resonances and at the resonances, of course, the damping is very important and shows quite different results. For very high frequencies (for example, f=10 cps and no resonance) the damping is of minor effect for the damping range which is of interest for the usually used propellants. The resonance irregularity is very sharp, i.e., the damping is of minor influence up to a very narrow interval at the resonance frequency.

For increasing acceleration, the fluid force and the moment is decreasing as can be seen from the previous formulae. The liquid height is of considerable influence on the fluid force and moment as shown in Fig. 5 and 6. For small forced frequencies, the damping plays a minor part. Near resonance, of course, the damping coefficient g is of strong influence on the fluid forces and moments. For forced frequencies smaller than the first resonance, but close to it, the fluid force decreases at a certain fluid height. For small damping, the decrease is stronger. For a certain fluid height near the maximum bending displacement, the fluid force part due to sloshing is great, while for higher liquid heights, this force is smaller due to smaller wall displacements. The additional force due to inertia of the additional fluid part is not great enough to overcome the difference of the sloshing forces for the two different heights. For higher damping, the slosh force difference is smaller and is overcome by the inertia forces, which means that there is no decrease in fluid force with increasing fluid height. For a forced frequency which is slightly higher than the first resonance, the fluid force has, for small damping, a considerable decrease with increasing fluid height and increases again. Higher damping shows the same effect up to the intersection point of all curves as mentioned for higher damping below the first resonance. Behind the intersection point, the fluid force will be greater for higher damping. For very high forced frequencies, the damping in the range of interest is of minor importance, and the force is increasing with increasing fluid height.

The fluid moment around the center of gravity of the undisturbed fluid due to bending tank walls is decreasing for increasing fluid height from a certain fluid height on for the same reason as mentioned before for the fluid force. For low forced frequencies, the damping is of less importance, while near resonances, it is of strong influence. For very high forced frequencies, the damping is of minor influence.

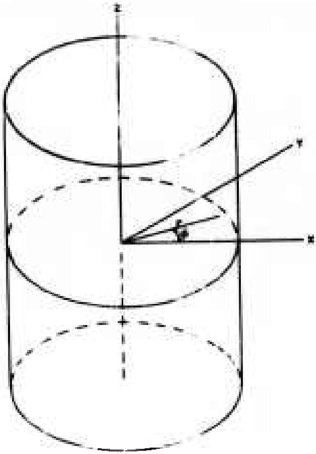
REFERENCES:

1. Helmut F. Bauer: "Fluid Oscillations in a Circular Cylindrical Tank" ABMA Report TR-1-58
2. Helmut F. Bauer: "Fluid Oscillation in a Cylindrical Tank with Damping" ABMA Report TR-4-58
3. Helmut F. Bauer: "Fluid Oscillations in a Circular Cylindrical Tank due to Bending of the Tank Wall" ABMA Report TR-3-58



$X_o(Z) = -\frac{4}{250}\left[\left(\frac{Z}{a}\right)^3 + \frac{Z}{a} - 6\right]$

BENDING TANN WALLS



COORDINATES

FIG. 1

FIG. 26(d) PRESSURE DISTRIBUTION DUE TO BENDING TANK WALLS.

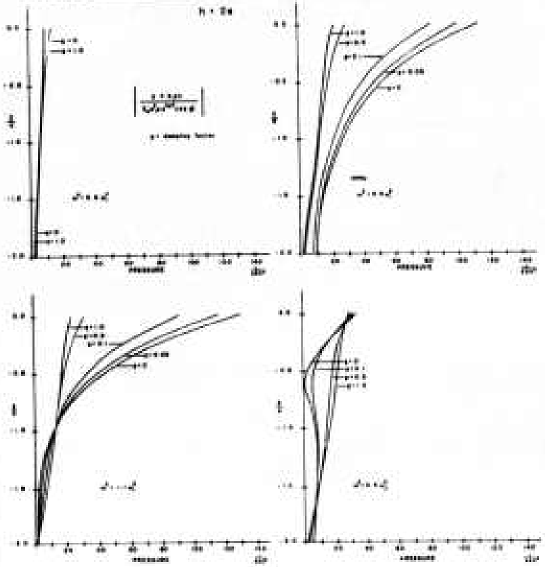


FIG. 2(b) PHASE ANGLE OF PRESSURE DISTRIBUTION DUE TO BENDING TANK WALLS

$h = 2a$

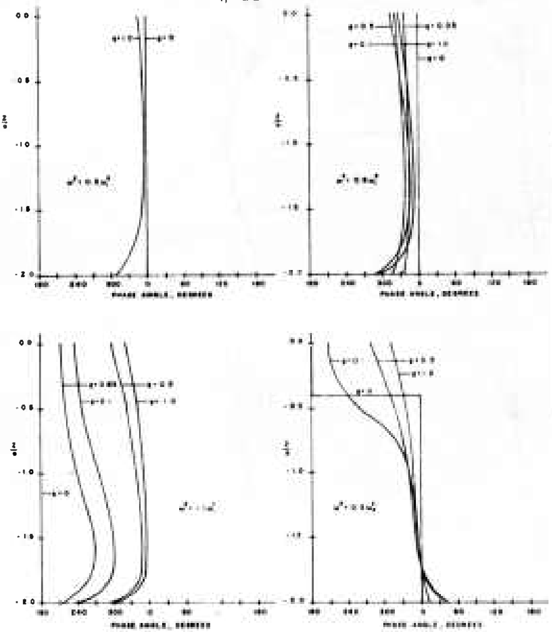


FIG. 3(B) PRESSURE DISTRIBUTION DUE TO BENDING TANK WALLS

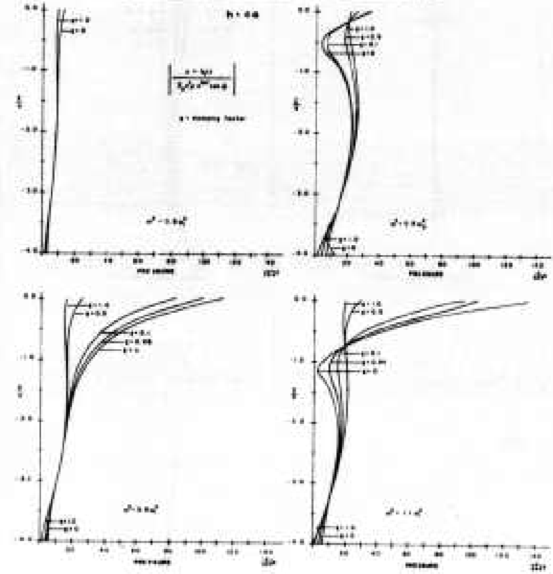


FIG. 3(b) PHASE ANGLE OF PRESSURE DISTRIBUTION DUE TO BENDING OF TANK WALLS

h = 4a

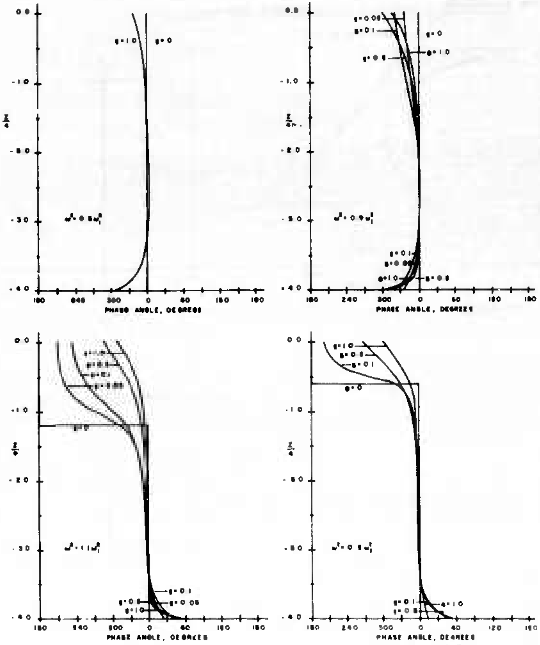
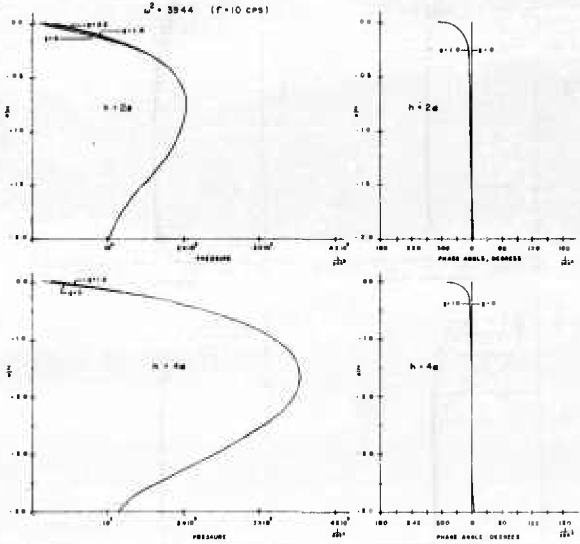


FIG. 4: PRESSURE DISTRIBUTION DUE TO BENDING TANK WALLS



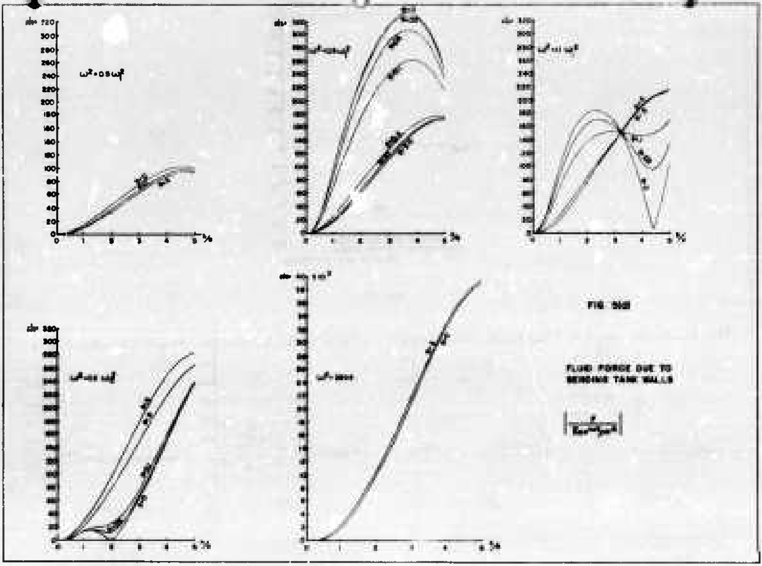


FIG.

FLUID FORCE DUE TO BENDING TANK WALLS

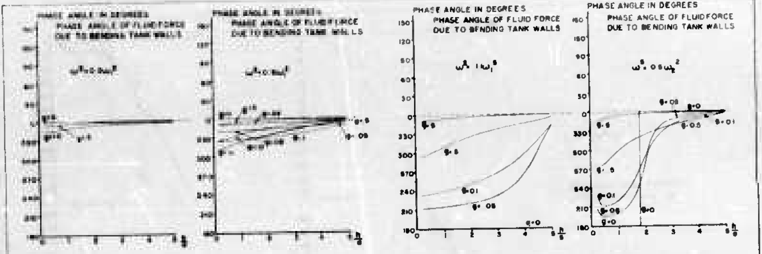
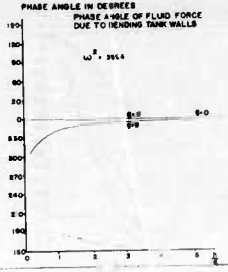
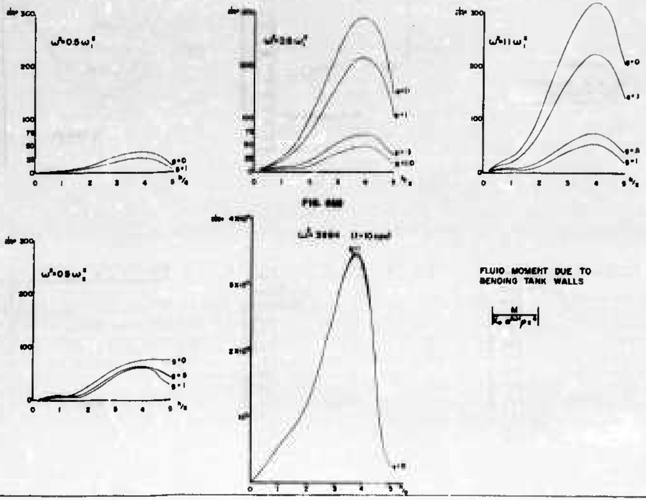
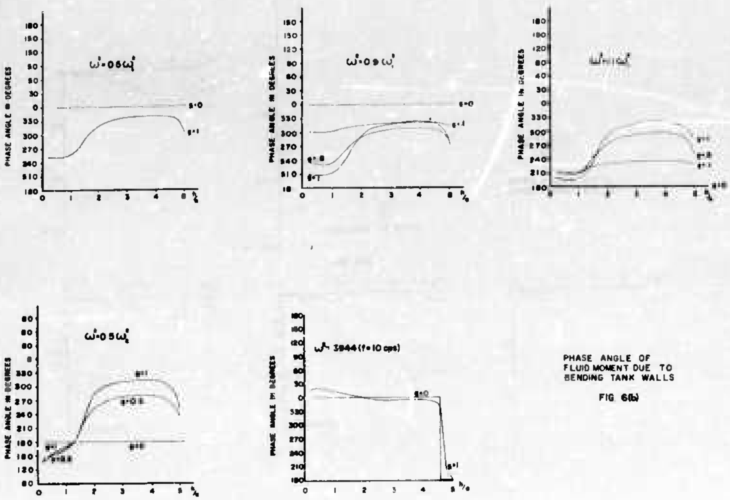

PHASE ANGLE IN DEGREES
PHASE ANGLE OF FLUID FORCE
DUE TO BENDING TANK WALLS
PHASE ANGLE IN DEGREES
PHASE ANGLE OF FLUID FORCE
DUE TO BENDING TANK WALLS

FIG 5(b)


PHASE ANGLE IN DEGREES
PHASE ANGLE OF FLUID FORCE
DUE TO BENDING TANK WALLS



FLUID MOMENT DUE TO BENDING TANK WALLS



PHASE ANGLE OF
FLUID MOMENT DUE TO
BENDING TANK WALLS

FIG 6(b)